3.1 Key Terms/Concepts:

Derivative of a Constant Function
Power Rule
Constant Multiple Rule
Sum/Difference Rule

3.1 Formulas—What does each mean?

$$\frac{d}{dx}(c)=0$$

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\frac{d}{dx}\left(f(x)\pm g(x)\right)=f'(x)\pm g'(x)$$

$$\frac{d}{dx}\left(cf(x)\right)=c\frac{d}{dx}\left(f(x)\right)=cf'(x)$$

Exercise #20 p. 181

Differentiate the function.

$$f(t)=\sqrt{t}-\frac{1}{\sqrt{t}}$$

Exercise #24 p. 181 (modified)

Differentiate the function

$$f(x)=\frac{x^2-2\sqrt{x}}{x}+4\pi^2$$

Exercise #54 p. 181

Find an equation of the tangent line to the curve $y=x\sqrt{x}$ that is parallel to the line $y=1+3x$.

3.2 Key Terms/Concepts:

The Product Rule

"One dee two plus two dee one"

The Quotient Rule

"Low dee high minus high dee low over low squared"

3.2 Formulas –What does each mean?

$$\frac{d}{dx}\left(f(x)g(x)\right) = f(x)g'(x) + g(x)f'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$$

Differentiate the following functions:

Exercise #14 p 187

$$f(x) = \frac{x+1}{x^3 + x - 2}$$

Exercise #12 p 187

$$R(t) = (t + e^t)(3 - \sqrt{t})$$

Exercise #26 p 188 (modified)

$$f(s) = \frac{as+b}{cs+d}$$

Exercise 19 & 22 p 188 (modified)

$$F(v) = \frac{\left(v^3 - 2v\sqrt{v}\right)\left(v - \sqrt{v}\right)}{2 + \sqrt{v} + v}$$

3.3 Formulas

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

Exercise #10 p. 195

Differentiate $y = \dfrac{1+\sin x}{x+\cos x}$

Exercise #16 p. 195

Differentiate $y = x^2 \sin x \tan x$

Exercise #42 p. 196

Compute the limit $\lim\limits_{\theta \to 0} \dfrac{\cos\theta - 1}{\sin\theta}$

Exercise

Differentiate

$$f(\alpha) = \frac{\cos\alpha \csc\alpha}{1 - \sec\alpha + \cot\alpha}$$

3.4 Key Terms/Concepts:

Chain Rule & Applications

"Derivative of the outer times the derivative of the inner"

3.4 Formulas –what does each mean?

$F(x) = f\left(g(x)\right) \Rightarrow F'(x) = f'(g(x)) \cdot g'(x)$

$\frac{d}{dx}\left(c^x\right) = c^x \ln c$

Exercise #16 p. 203

Differentiate $y = 3\cot(n\theta)$

Exercise #20 p. 203

Differentiate $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$

Exercise #26 p. 204

Differentiate $G(y) = \frac{(y-1)^4}{(y^2 + 2y)^5}$

Exercise #52 p. 204

Find an equation of the tangent line to the curve at the given point:

$y = \sin x + \sin^2 x;\ (0,0)$

3.5 Key Terms/Concepts:
Implicit Differentiation
Derivatives of Inverse Trig Functions

3.5 Formulas

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2+1}}$$

$$\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{x\sqrt{x^2+1}}$$

$$\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$$

Exercise #48 p. 214
Find the derivative of the function. Simplify as much as possible.

$$y = \sqrt{x^2-1}\,\sec^{-1} x$$

Exercise #14 p. 213
Find *dy/dx* by implicit differentiation

$$y\sin(x^2) = x\sin(y^2)$$

Exercise #10 p. 213
Find *dy/dx* by implicit differentiation

$$y^5 + x^2y^3 = 1 + ye^{x^2}$$

Exercise #16 p. 213
Find *dy/dx* by implicit differentiation

$$\sqrt{x+y} = 1 + x^2y^2$$

3.6 Key Terms/Concepts:
Derivative of Log base a
Derivative of Natural Log
******LOG LAWS***
***Remember the CHAIN RULE

3.6 Formulas –what does each mean?

$$\frac{d}{dx}\left(\log_a x\right)=\frac{1}{x\ln a}$$

$$\frac{d}{dx}\left(\ln x\right)=\frac{1}{x}$$

Exercise #18 p. 220
Differentiate the function

$$H(z)=\ln\left(\sqrt{\frac{a^2-z^2}{a^2+z^2}}\right)$$

Exercise #22 p. 220
Differentiate the function

$$y=\log_2\left(e^{-x}\cos\pi x\right)$$

Exercise #38 p. 220
Use logarithmic differentiation to find the derivative of the following function

$$y=\sqrt{x}e^{x^2}\left(x^2+1\right)^{10}$$

Exercise #50 p. 220
Find y' if $x^y=y^x$

3.7 Key Terms/Concepts:
Average Rate of Change
Instantaneous Rate of Change
Velocity
Acceleration
Optional: Cost Function, laminar flow, current, compressibility

3.7 Formulas

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$a(t) = v'(t) = s''(t)$$

Section 3.7 #15 p. 231

A spherical balloon is being inflated. Find the rate of increase of the surface area ($S = 4\pi r^2$) with respect to the radius *r* when *r* is (a) 1 ft., (b) 2 ft., and (c) 3 ft. What conclusion can you make?

Section 3.7 #18 p. 231

If a tank holds 5,000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume *V* of water remaining in the tank after *t* minutes as

$$V = 5000\left(1 - \frac{t}{40}\right)^2 \quad 0 \le t \le 40$$

Find the rate at which water is draining from the tank after (a) 5 min (b) 10 min (c) 20 min (d) 40 min

Section 3.7 #30 p. 233

The cost function for production of a commodity is $C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3$

(a) Find and interpret $C'(100)$.

(b) Compare $C'(100)$ with the cost of producing the 101^{st} item.

3.8 Key Terms/Concepts:
Law of Natural Growth/Decay
Half-Life
Optional: Continuously Compounded, Newton's Law of Cooling

3.8 Formulas –what does each mean?

$$\frac{dy}{dt} = ky$$

$$y(t) = y(0)e^{kt}$$

Section 3.8 #1 p 239

A population of protozoa develops with a constant relative growth rate of 0.7944 per member per day. On day zero the population consists of two members. Find the population size after six days.

Section 3.8 # 4 p. 239-240

A bacteria culture grows with a constant relative growth rate. After 2 hours there are 600 bacteria and after 8 hours the count is 75,000.
(a) Find the initial population
(b) Find an expression for the population after *t* hours.
(c) Find the number of cells after 5 hours.
(d) Find the rate of growth after 5 hours.
(e) When will the population reach 200,000?

Section 3.8 #10 p. 240

A sample of tritium-3 decayed to 94.5% of its original amount after a year.
(a) How long is the half-life of tritium-3?
(b) How long would it take the sample to decay to 20% of its original amount?

3.9 Key Terms/Concepts:

Related Rates Process:

1. Read the problem *carefully*
2. Draw a picture
3. Convert everything into math lingo – derivatives, etc.; note which quantities are time dependent
4. Write an equation that relates all the quantities given in the problem. Use geometry of problem to eliminate unknown quantities.
5. Use chain rule to differentiate each side with respect to (w.r.t.) time
6. Substitute given info into this differentiated equation.

Exercise 1

Two cars are traveling on long straight roads that meet at right angles. Car A leaves the intersection traveling east at 48 mph and car B leaves the intersection 3 hours later and travels north at 50 mph. At what rate is the distance between the two cars increasing 2 hours after car B leaves the intersection?

Exercise 2

A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 4 ft/sec. How rapidly is the area enclosed by the ripple increasing at the end of 8 seconds?

Exercise 3

Sand is being dumped from a conveyor belt at the rate of $18\pi\ ft^3/\min$. The coarseness of the sand is such that it forms a pile in the shape of a cone with the radius of the base always 1/3 the height. How fast is the height increasing when the pile is 15 feet high?

3.10, 4.8 Key Terms/Concepts:

Linear approximation
Linearization
Differentials
Relative Error

Newton's Method

3.10, 4.8 Formulas –what does each mean?

$f(x) \approx L(a) = f(a) + f'(a)(x-a)$

$\Delta y = f(x+\Delta x) - f(x)$

$R.E. = \frac{\Delta y}{y}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Exercise #14 p. 252

Find the differential of each function.

(a) $y = e^{tan(\pi t)}$

(b) $y = \sqrt{(1+\ln z)}$

Exercise #28 p. 252

Use a linear approximation (or differentials) to estimate the given number.

$\sqrt{99.8}$

Exercise #14 p. 338

Use Newton's method to approximate the indicated root of the equation to six decimal places: The root of $2.2x^5 - 4.4x^3 + 1.3x^2 - 0.9x - 4.0 = 0$ in the interval $[-2,-1]$.